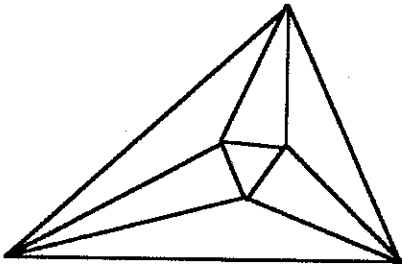


Minnesota State High School Mathematics League

Individual Event



2005-06 Event 5A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 20 minutes for this event.

Faces Painted
Number of cubes

1	
2	
3	

1. A cube 4 inches on its edges is painted red. Then, by making cuts parallel to the faces, it is cut into 64 cubes with 1 inch edges. Fill in the table that counts the number of cubes having 1, 2, or 3 faces painted.

2. In the two additions,
- $$A + B = C$$
- $$C + D = EA$$

each of the five letters represents a distinct digit, EA being a two digit number. What is the value of $B+D$?

3. With one straight line, you can slice a pie into two pieces; a second cut that crosses the first one will produce four pieces; and a third cut can produce as many as seven pieces (Figure 3). What is the largest number of pieces that you can get with seven straight cuts?

4. (a) Amy, Beth, and Christine toss a coin 15, 16 and 17 times respectively. Which girl is least likely to get more heads than tails?

- (b) Amy, Beth, and Christine toss a coin 18, 19 and 20 times respectively. Which girl is least likely to get more heads than tails?

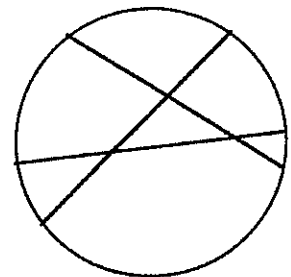
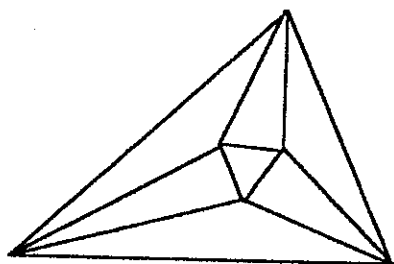


Figure 3

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2005-06 Event 5B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. An isosceles trapezoid has base angles of 60° and bases of 14 and 8. What is its area?
- _____ 2. $\triangle ABC$ has sides of lengths $a = 4$, $b = 10$, and $c = 7$. $\triangle UVW \sim \triangle ABC$, and $\triangle UVW$ has a perimeter of 15. What is the length of the longest side of $\triangle UVW$?
- _____ 3. $ABCDEF$ is a regular hexagon with center P and sides of length s . $\triangle PQR$ is an equilateral triangle with Q and R exterior to the hexagon as shown in Figure 3. PR intersects AB at U , making $AU = \frac{s}{3}$. Find in terms of s the area of the quadrilateral common to both the triangle and the hexagon.
- _____ 4. In $\triangle ABC$ (Figure 4), $AB = 9$, $AC = 15$, and $m(\angle B) = 2m(\angle C)$. What is the area of $\triangle ABC$?

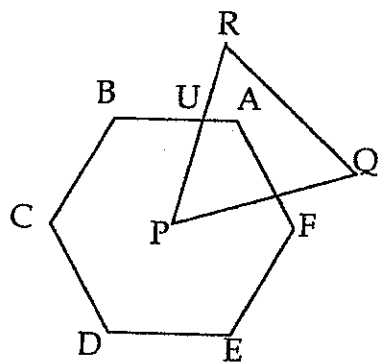


Figure 3

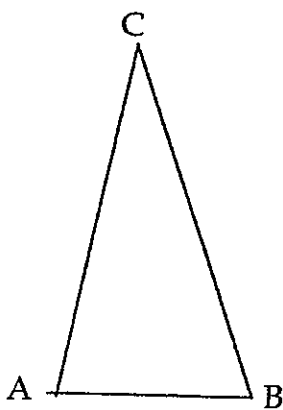
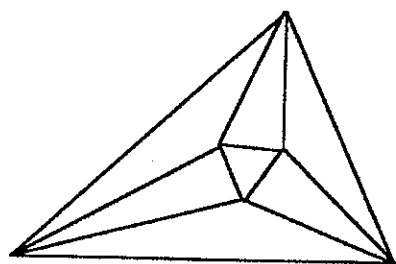


Figure 4

Name _____ Team _____



Minnesota State High School Mathematics League

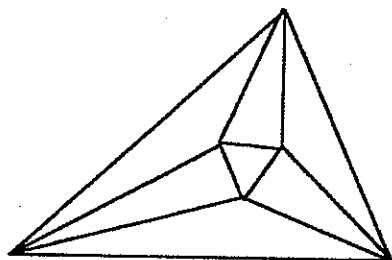
Individual Event

2005-06 Event 5C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. In the expansion of $\left(\frac{1}{2}a + 4\right)^8$, what will be the coefficient of a^5 ?
- _____ 2. If you draw three cards from an ordinary 52 card deck, what is the probability of drawing at least one of the twelve face cards (i.e. a Jack, Queen, or King)?
- _____ 3. Five pennies made in 2000, 2002, 2004, 2005, and 2006 are to be put in a pile, face up. In how many ways can this be done so that the 2005 and 2006 pennies are not touching each other?
- _____ 4. Sarah was sent to get 8 cans of soda to have on hand for the study session. When she got to the machine, she found that she had six choices. Remembering that she had a die in her purse for some homework for her probability class, she decided to roll it 8 times, choosing the first flavor if a 1 came up, etc. for all six possibilities. Using this scheme, what is the probability that she gets exactly four diet cokes?

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2005-06 Event 5D

Questions in this event are written, with permission, as variations of problems from the 2005 AMC-12 Exam. Review of this exam is excellent preparation, not only for this event, but for the 2006 AMC-12, which we strongly encourage you to take.

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

A = 1. A and B are digits such that $(100A + 10A + B)(2B + 1) = 2006$. Find A and B .
B =

2. The mean average of the eighteen numbers in the set $A = \{x_1, \dots, x_{18}\}$ is 12.
 The mean average of the thirty numbers in the set $B = \{y_1, \dots, y_{30}\}$ is 20.

What is the mean average of the forty-eight numbers in the set $A \cup B$?

3. A lattice point is a point having integers for coordinates. The point $(4,5)$ is, for instance, a lattice point that lies on the line $7x - 11y + 27 = 0$. What is the total number of lattice points on this line that lie in the square

$$S = \{(x, y) : 0 \leq x \leq 100, 0 \leq y \leq 100\}$$

4. Given: $f(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 4 - 2x & 1 \leq x \leq 2 \end{cases}$

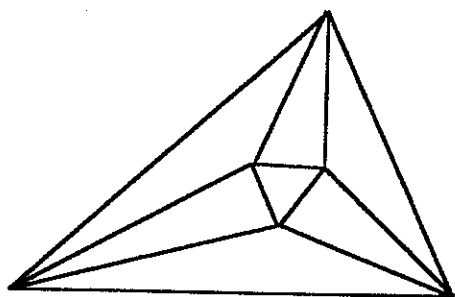
$$g(x) = f(f(x))$$

$$h(x) = g(g(x))$$

Find $h\left(\frac{7}{8}\right) + h\left(\frac{1}{8}\right)$

Name _____

Team _____



Minnesota State High School Mathematics League

Team Event

2005-06 Meet 5

a	b	c
50	25	375

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

1. A set of integer solutions to $a^3 + b^3 = c^2$ is given by $a = 50$, $b = 25$, and $c = 375$. There are other sets of solutions, some using smaller integers. You get 1 point, up to a maximum of 4, for each distinct set of integer solutions, none of the integers 0, that you obtain different from the one given.
2. How many integers are there between 100,000 and 1,000,000 that, like 134,789, consist of distinct digits that increase from left to right?
3. An equilateral $\triangle ABC$ has sides of length 3. Point P is located interior to the triangle at a distance 1 from AB and a distance 1 from AC . Correct to three places to the right of the decimal, what is the distance of P from BC ?
4. Expressing numbers to base ten (i.e. no trickery here), how many 1's will appear in the integer which is the sum of the ninety-nine integers $9 + 99 + 999 + \dots + 99\dots9$
5. Albert, Bob and Charles play a game in which there is one clear loser. Starting with different amounts of money, they decide that the loser is to pay to each of the others an amount that doubles each winners money. After three games each man has lost exactly once, and each has \$24. Albert had the most money when they started. How much?
6. A line tangent to $x^2 + y^2 = r^2$ in the first quadrant has a slope of m . What, in terms of r and m , is the area of the triangle that this line forms with the positive coordinate axes?

Team _____